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Chapter 12

Vertical Alignment

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Chapter 12 Amendments - July 2002

Revision Register

Issue/ Rev No.	Reference Section	Description of Revision	Authorised by	Date
1		First Issue	Steering Committee	May 2000
2	12.3.3	Modification to Equation 12.3	W. Semple	Feb 2001
3	12.1	Additional paragraph	Steering Committee	July 2001
	12.3.3	Addition to note in Table 12.5. Additional paragraph in “Sight Distance - Headlight”		
	12.5	New Section 12.5		
	12.7	Old Section 12.5. Subsequent sections renumbered		
	New	Relationship to other chapters		
4	12.2	Cross reference - Reference to Fig. 15.1 at end of section	Steering Committee	Feb 2002
	12.2.1	Maximum grades - Table 12.2 modified - remove bracketed comment		
	12.2.2	Length of grades - Additional discussion and inclusion of MCV requirements		
	12.2.3	Steep grades - Minor amendments		
	12.2.4	Minimum grades - Further discussion of drainage requirements		
	12.3.3	Crest VC's - Further discussion of treatment of sub-standard VC's on widen and overlay projects		
	12.3.3	Reaction times - Reaction time of 1.5secs removed. Therefore, amendments to Table 12.5 required; additional note under Table 12.5		
	12.3.4	Sag VC's appearance - Further comments on the design of short Sag VC's		
	12.3.4	Headlight sight distance - Direction of headlight beam of 1° upwards adopted - extensive changes to Table 12.6 included; reaction time of 1.5secs removed		
	12.3.6	Asymmetric VC's - Cross reference to Appendix 12B added for asymmetric VC's		
	12.5	Editing - Editing changes to previous amendment.		
	12.6	Hidden dip grading - Additional comment on assessment of hidden dip		
5	12.3.4	Amendments to Tables 12.5 and 12.6. Additional paragraph in “Sight Distance - Headlight”.		

Chapter 12
Vertical Alignment

12.1 General

Vertical alignment (referred to as, and geometrically represented by the grade line or longitudinal section) consists of straight grades joined by vertical curves. The final design location of the vertical alignment is principally the fit to the natural terrain but consideration also needs to be given to the maximum allowable grades, the minimum radius vertical curves, the economical balancing of the earthworks, the appearance, property acquisition, environmental impacts, and the co-ordination with the horizontal alignment.

The design criteria which dominate in deciding on the appropriate alignment vary with the type of road being considered. On minor roads, the economy of the design and the impacts on adjacent property dominate over appearance, but the desirability of obtaining a good appearance should not be discounted. It often does not cost any more to achieve a good appearance when the appropriate steps are taken in the first instance. On major roads, appearance takes on a greater role and will tend to dominate over the earthworks balance requirements but the need to provide an economically sound design cannot be ignored. It may be possible with good design and appropriate coordination with the horizontal alignment to achieve all of the criteria.

On undivided roads, the vertical alignment is designed as the surface of the pavement along the construction centre line. On divided roads, the vertical alignment is usually represented by the line along the lane edge against the median.

On divided roads with independent grading of the carriageways, the vertical alignment is usually designed as the surface of the pavement along the construction centre line of each carriageway.

The vertical alignment must be designed in conjunction with the horizontal alignment and both coordinated in accordance with the principles stated in Chapter 10.

12.2 Grades

Generally, grades should be as flat as possible consistent with economy. Flat grades permit all vehicles to operate at the same speed. Steeper grades produce variation in speeds between lighter vehicles and the heavier vehicles both in the uphill and downhill directions. This speed variation leads to higher relative speeds of vehicles producing the potential for higher rear-end and head-on vehicle accident rates. This speed variation also results in increased queuing and overtaking requirements which give rise to further safety problems particularly at higher traffic volumes. In addition, freight costs are increased due to the slow speed of heavy vehicles.

Table 12.1 shows the effect of grade on vehicle performance and lists road types which would be suitable for these grades. (See also Figure 15.1.)

12.2.1 Maximum Grades

Table 12.2 shows general maximum grades over long lengths of road for a range of typical situations.

The following situations may justify the adoption of grades steeper than the general maximum:

- comparatively short sections of steeper grade which can lead to significant cost savings.
- difficult terrain in which grades below the general maximum grades are not practical.
- absolute numbers of heavy vehicles are generally low.
- less important local roads where the costs of achieving higher standards are less able to be justified.

The first three conditions will apply even on major roads.

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Table 12.1 Effect of Grade on Vehicle Type

Grade	Reduction in Vehicle Speed as compared to Flat Grade				Road Type Suitability
	Uphill		Downhill		
	Light Vehicle	Heavy Vehicle	Light Vehicle	Heavy Vehicle	
0-3	Minimal	Minimal	Minimal	Minimal	For use on all roads.
3-6	Minimal	Some reduction on high speed roads	Minimal	Minimal	For use on low-moderate speed roads (incl. high traffic volume roads)
6-9	Largely unaffected	Significantly slower	Minimal	Minimal for straight alignment. Substantial for winding alignment	For use on roads in mountainous terrain. Usually need to provide auxiliary lanes if high traffic volumes
9-12	Slower	Much slower	Slower	Significantly slower for straight alignment. Much slower for winding alignment	Need to provide auxiliary lanes for moderate - high traffic volumes. Need to consider run-away vehicle facilities if the number of commercial vehicles is high
12-15	10 -15 km/h slower	15% max. negotiable	10 -15 km/h slower	Extremely slow	Satisfactory on low volume roads (very few or no commercial vehicles)
15-33	Very slow	Not negotiable	Very slow	Not negotiable	Only to be used in extreme cases and be of short lengths (no commercial vehicles)

Note: Grades over 15% should be used only in extreme cases (eg. access to an isolated vantage point) and should be for short lengths where no heavy vehicles are required to use them.

Table 12.2 General Maximum Grades (%)

Target Speed (km/h)	Terrain					
	Flat	Rolling	Mountainous			
			<2000 AADT	2000-5000 AADT (See Note 1)	5000-10000 AADT (See Note 2)	>10000 AADT (See Note 3)
50	6-8	8-10	12	10	-	-
60	6-8	7-9	10	9	9	-
80	4-6	5-7	9	8	7	7
100	4	4-6	-	-	6	6
120	3	3-5	-	-	-	5

NOTES:

1. Overtaking lanes may be required depending on overtaking opportunities and percentage commercial vehicles.
2. Overtaking lanes would be recommended where overtaking opportunities are occasional and the component of commercial vehicles is greater than or equal to 5%.
3. Four lanes divided or undivided should be investigated.
4. The maximum design grade should be used infrequently, rather than as a value to be adopted in most cases.

12.2.2 General Maximum Lengths of Steep Grade

It is undesirable to have a very long length of steep grade. On both the upgrade and downgrade, the lower operating speed of trucks may cause inconvenience to other traffic. When it is impracticable to reduce the length of grade to the desirable length, consideration should be given to providing an auxiliary lane both in the uphill and downhill directions. (See Chapter 15 for details).

Current vehicle standards require multi-combination vehicles to be able to maintain a speed of 70 km/h on a 1% grade. The length of grade plays an important factor in the performance of those vehicles.

Suggested desirable maximum length of grades not requiring special consideration are given in Table 12.3.

For sections of road with grades greater than those given in Table 12.3, a risk analysis to identify likely effects should be carried out to determine the most appropriate treatment.

Table 12.3 Desirable Maximum Lengths of Grades

Grade (%)	Length (m)
2-3	1800
3-4	900
4-5	600
5-6	450
>6	300

It is a worthwhile design feature to avoid horizontal curves, or at least sharp curves, at the bottom of steep grades, where speeds may develop to the point of difficult vehicle control. This is likely only in the case of 85th percentile speeds greater than 60 km/h.

Computer simulation (VEHSIM) may be used to access truck performance on grades. It should be noted that trucks can reach excessive speeds on long downhill grades as low as 3% when out of control (NRTC, 1997). Providing escape facilities (Chapter 15) may be required for these cases.

12.2.3 Steep Grade Considerations

Although speeds of cars may be reduced slightly on steep upgrades, large differences between speeds of light and heavy vehicles will occur and speeds of the latter will be quite slow. It is important, therefore, to provide adequate horizontal sight distance to enable faster vehicle operators to recognise when they are catching up to a slow vehicle and to adjust their speed accordingly.

On any generally rising or generally falling section of road, adverse grades (grades opposite to the general rise or fall of the section) should be avoided as much as practicable, as they are wasteful of energy. Where possible, it is preferable to introduce a flatter grade at the top of a long ascent, particularly on low speed roads, but this must not be forced by such an expedient as steepening the lower portion of the grade.

On steep downgrades, it is desirable to increase the design speed of the individual geometric elements progressively towards the foot of the steep grade. Where this cannot be achieved and where percentages of heavy vehicles are high, consideration should be given to construction of runaway vehicle facilities. These usually take the form of an upward sloping escape ramp or an arrestor bed of sand or gravel. Refer to Chapter 15 for warrants and details of runaway vehicle facilities.

For further discussion on speeds on steep grades, see Chapter 6, Section 6.2.1.

12.2.4 Minimum Grades

Very flat grades may make it difficult to provide longitudinal drainage in table drains, kerb and channel and medians, where these parallel the road grade. As far as possible, these drainage requirements should not dictate the road grade, rather the drainage facility should be designed to accommodate the road grade. This may require greater recourse to sub-surface drains with closely spaced inlets, or independently graded table drains, or other solutions to suit the circumstances.

Care should be taken in cases where a flat grade is combined with horizontal curvature. The rotation of the pavement may create a situation where the flow path crosses from one side of a lane to the other, resulting in undesirable depths of water on the pavement surface.

Worse conditions can occur on steep grades combined with successive curves in opposite directions. The combination of grade and pavement rotation can create a situation where the flow path meanders from one side of the road to the other with the depth of flow becoming excessive.

In both of those cases, pavement contours should be examined to ensure these conditions do not exist. If found to exist, action to change the parameters (e.g. increasing the rate of rotation) must be taken to control the condition.

- **Cuttings**

  Generally, the minimum grade in cuttings is 0.5% to allow adequate fall in unlined drains. However, it is permissible to provide flatter grades provided that a minimum grade of 0.5% is retained in the unlined drains. This is done by uniformly widening the drains at their standard slope, thereby deepening them progressively, or alternatively, lining the table drains will permit a flatter grading of table drains to be adopted. In constrained situations, the slope of the drains may be reduced below 0.5% provided that they are lined.

- **Medians**

  On divided roads the necessity for median drainage may control the minimum roadway grade. However, where very flat roadway grades are the only practical solution, sag gullies at regular intervals with cross drainage may be provided in the median. This will enable median slopes greater than the roadway slope to be used. The grade considerations given for cuttings also apply to medians.

- **Kerb and Channel**

  Where a kerb and channel forms part of the type cross section, for example, a median kerb on a divided road, the minimum grade of the channel formed by the pavement edge and kerb should not be less than 0.5% except in difficult circumstances, when the absolute minimum shall be 0.2%. This usually influences the grade of the pavement. Where this situation exists on a long flat section of the road, care must be taken to avoid a wavy grade line, by making grade changes either well spaced or at the start or end of horizontal curve, so as to disguise the changes of grade.

  The use of kerb and channel on high speed alignments is generally undesirable (except in constrained situations and prior to intersections). In addition to the safety problems caused by the kerb, separate drainage facilities are required. In these cases, a shallow concrete lined v-drain with subsoil drainage usually provides a better solution.

12.3 Vertical Curves

12.3.1 Curve Geometry

Generally, the type of vertical curve used is a parabolic curve with the following formula:

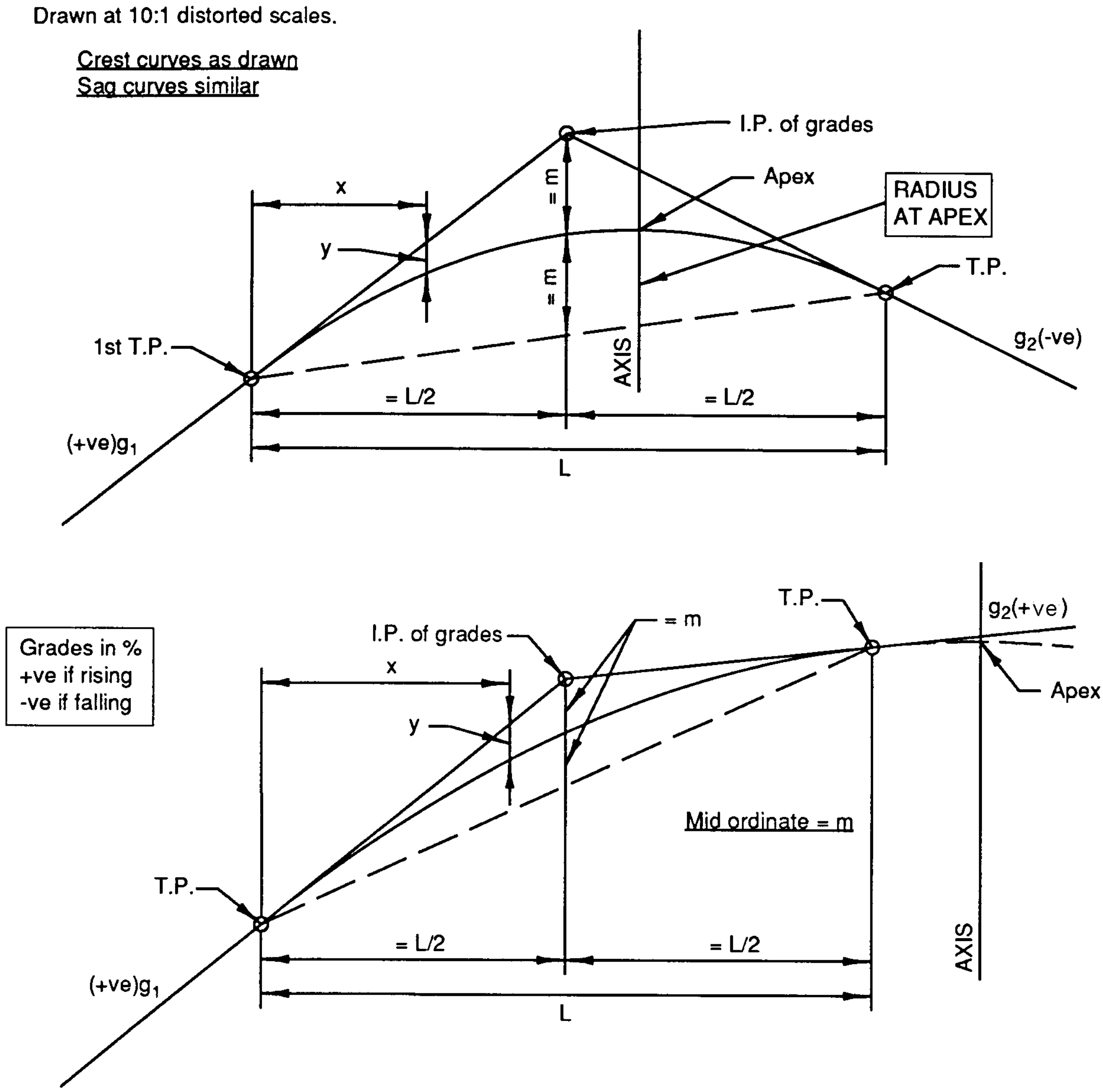
$$y = k\,x^2 \qquad (12.1)$$

where y = vertical offset
k = a constant
x = horizontal distance

Figure 12.1 details the above parameters. There are three associated parameters which are radius, length, and grade difference and their correlation is also shown on Figure 12.1.

Traditionally, the parabola has been used because of its simplicity and because all formulae are exact whereas the same formulae used with the circle would be approximate. When drawn to the distorted scales of the longitudinal section (usually 10:1) the curve is distinctly parabolic but a circle to the same scale would also look parabolic.

The vertical offsets ‘y’ from a tangent are proportional to the square of the distances ‘x’ measured horizontally from the tangent point to the offset point. Being parabolic does not mean that the curve has any transitional properties. That portion of the parabola within the grade angles is so close to a circle that the difference is negligible.



Length equally spaced horizontally about I.P.

$$y = kx^2 = \frac{(g_2 - g_1)x^2}{200L} \text{ - (1)}$$

$$y = \frac{x^2}{2R} \text{ - (3)}$$

where

- y = offset of parabola down from grade line (m)
- x = horizontal distance from tangent point to the offset point (m)
- g_1 = 1st grade (%)
- g_2 = 2nd grade (%)
- L = length of horizontal projection of the arc

$$m = \frac{L(g_2 - g_1)}{800} \text{ - (4)}$$

where

- m = mid ordinate (m)

$$R = \frac{100L}{(g_2 - g_1)} \text{ - (2)}$$

where

- R = equivalent radius of vertical curve (m)

Figure 12.1 Typical Vertical Curves

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The length of curve is not the length of arc but the horizontal projection of the arc. The axis of the parabola is always vertical.

A common parameter used to define the size of the vertical curve is the equivalent radius ‘R’. Radius is used in this manual to designate the limiting curvature required for a given design speed. Radius is a single number and is a constant for the curve irrespective of the grades and length of the curve.

Appendix 12A shows the derivation of the various vertical curve formulae. Solutions for some of the situations arising with vertical curves are also provided.

Appendix 12B shows the derivation of the formulae for asymmetric vertical curves. These are not generally used, but there may be circumstances where it is necessary and convenient to adopt them.

12.3.2 Minimum Radii of Vertical Curves

Generally, the largest radii vertical curves should be used provided they are reasonably economical. However in difficult situations, vertical curves approaching the minimum may be considered where the costs of providing larger curves makes their use prohibitive.

There are three controlling factors affecting the selection of the minimum radius vertical curve - the appearance, the riding comfort and the sight distance.

12.3.3 Crest Curves

For a particular design speed, the minimum radius crest vertical curve is usually governed by sight distance requirements. However, for small changes of grade, the appearance criterion may suggest larger values of radius to provide satisfactory appearance of the curve. Riding comfort is generally not considered on crests as the sight distance requirements almost always require the use of large radius vertical curves.

Although vertical crest curves as large as possible are generally used, there are cases where the provision of a sharper crest vertical curve with a longer vertical straight will lead to longer and safer overtaking opportunities. This issue becomes important on high speed roads with limited overtaking opportunities caused by tight alignments or traffic density.

Projects often seek to widen and overlay existing pavements on the existing alignment. Where the existing alignment includes crest vertical curves with radii too short for the design speed, it may be possible to retain the existing vertical alignment by adopting the Manoeuvre Sight Distance as the criterion and widening the sealed pavement (see 9.3.1).

If the required radius is still greater than the existing, regrading will be required to achieve manoeuvre sight distance. Designers should examine alternative arrangements for the vertical curves and grades to minimise the extent of earthworks.

In all cases where the manoeuvre sight distance is used, the pavement must be widened to provide the room required for the evasive action. Designers should also consider the appropriate object height to use i.e. what could be considered to be a probable case, e.g.

- dead animal (0.2m)
- stalled vehicle (0.6m tail light).

Appropriate signing should also be used in conjunction with this reduced standard (MUTCD).

It must be noted that the action of widening the seal and improving the surface will increase the 85th percentile speed on the road concerned.

Appearance

At very small changes of grade, a vertical curve has little influence other than appearance of the profile and although not desirable, may be omitted. At any significant change of grade, short vertical curves detract from the appearance.

Table 12.4 gives minimum vertical curve lengths for satisfactory appearance. Longer curves may be preferred where they can be used without conflict with other design requirements, e.g. overtaking, and where they give better fit to the topography.

The values in Table 12.4 are subjective approximations and therefore the lack of precision is intentional. General ranges, not precise values are relevant.

Sight Distance

Where the length of vertical crest curve is greater than the sight distance, the minimum radius crest vertical curve is given by Equation 12.2.

$$R = \frac{D^2}{2(\sqrt{h_1} + \sqrt{h_2})^2} \qquad (12.2)$$

where

R = minimum radii of crest curve (m)

D = sight distance required (m)

h_1 = height of eye above road or headlight height (m)

h_2 = object cut-off height above road (m)

For the various configurations of D, h_1 and h_2, refer Chapter 9 Sight Distance. An example of the use of the above calculation is given in Example 12.1.

Minimum radii crest vertical curves (where the vertical curve length exceeds the sight distance) are shown in Table 12.5 for various combinations of design speed, type of sight distance, roadway type, reaction times and vertical height parameters.

Where the length of vertical crest curve is less than the sight distance, the minimum radius crest vertical curve is given by Equation 12.3.

Note that the crest vertical curves are not generally designed for overtaking sight distance. The resulting curve is generally too long for practical purposes. However, long crest curves may be appropriate for appearance reasons on curvilinear alignments on flat plains. These should be designed on their merits for each case.

$$R = \frac{200D}{A} - \frac{20000(\sqrt{h_1} + \sqrt{h_2})^2}{A^2} \qquad (12.3)$$

where

R = minimum radius of crest curve (m)

D = sight distance required (m)

A = change of grade (%)

h_1 = height of eye above road or headlight height (m)

h_2 = object cut-off height above road (m)

For the various configurations of D, h_1 and h_2, refer Chapter 9 Sight Distance.

12.3.4 Sag Curves

Sag curves are generally designed as large as economically possible using the comfort criterion as a minimum. Usually, provision of the headlight sight distance criteria can only be practically applied on unlit high standard roadways in flat or partially rolling terrain.

Table 12.4 Length of Vertical Curves - Appearance Criterion

Design Speed (km/h)	Maximum Grade Change Without Vertical Curve (%)*	Minimum Length of Vertical Curve for Satisfactory Appearance (m)
40	1.0	20 - 30
60	0.8	40 - 50
80	0.6	60 - 80
100	0.4	80 - 100
120	0.2	100 - 150

* In practice vertical curves are frequently provided at all changes of grade.

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Table 12.5 Minimum Radius Crest Vertical Curves - Sight Distance Criteria (Length of Crest Curve Greater than the Sight Distance)

Design Speed km/h	Type C1 Restrictions to Visibility h_1=1.15m h_2=0.2m			Type C2 Restrictions* to Visibility h_1=0.75m h_2=0.2m		
	Manoeuvre Sight Distance (m)	Stopping Sight Distance (m)		Manoeuvre Sight Distance (m)	Stopping Sight Distance (m)	
	R_T= 2.0s	R_T= 2.5s	R_T= 2.0s	R_T= 2.0s	R_T= 2.5s	R_T= 2.0s
50	440		440	590		590
60	780		900	1000		1200
70	1200		1600	1600		2100
80	2000	2900	2400	2600	3800	3200
90	3100	4200	3700	4200	5700	4900
100	5200	6300		7000†	8400†	
110		9500†			13000†	
120		14000†			18000†	
130		19000†			26000†	

* Values in these columns only to be used where economically justified.

† The sight distance used to calculate these radii are greater than the range of most headlights (i.e. 120-150m). However, the minimum radius curves for C1 restrictions (2.5 sec R_T) will provide for headlight visibility to the tail light of a vehicle in the lane ahead.

NOTES:

1. Values of sight distance used for calculation of minimum radii crest curves are from Chapter 9 Sight Distance.
2. Those values not shown are generally not used.
3. **Where intersections are on a crest, curve radii must be sufficient to provide Safe Intersection Sight Distance (see Chapter 13).**

Appearance

Short sag curves can be perceived as a “kink”, and while safe and comfortable, will not be attractive. (See also Chapter 10.) This applies to all situations but may be exacerbated where the change in grade is small.

At small changes of grade, minimum lengths of sag vertical curves are to be in accordance with Table 12.4.

In other cases, a check on the appearance should be undertaken using perspective views. Chapter 10 also provides guidance. It is often possible to achieve a much larger radius sag curve for little or no extra cost.

Comfort

Discomfort is felt by a person subjected to rapid changes in vertical acceleration. To minimise such discomfort when passing from one grade to another, it is usual to limit the vertical acceleration generated on the vertical curve to a value less than 0.05g where g is the acceleration due to gravity. On low standard roads, at intersections or where economically justified, a limit of 0.10g may be used.

The minimum sag vertical curve radius for comfort can be calculated by Equation 12.4.

$$R = \frac{V^2}{12.96a} \qquad (12.4)$$

where

R = Sag curve radius (m)

V = Velocity (km/h)

a = Vertical acceleration (m/s/s)

Values of minimum radii for sag curves for specific design speeds and vertical accelerations of 0.05g and 0.10g are shown in Table 12.6. Since this is a subjective criterion, values have been rounded and should not be regarded as precise requirements.

Sight Distance

Headlight

Sight distance on sag curves is not restricted by the vertical geometry in daylight conditions or at night with full roadway lighting unless overhead obstructions are present. Under night conditions on unlit roads, as discussed in Chapter 9 Sight Distance, limitations of vehicle headlights on high beam restrict sight distance to between 120–150m for modern vehicles. On high standard roads not likely to be provided with roadway lighting, consideration may be given to providing headlight sight distance.

Nevertheless, where horizontal curvature would cause the light beam to shine off the pavement (assuming 3 degrees lateral spread each way), little is gained by flattening the sag curves.

In all cases adequate sight distance to the tail lights must be provided.

For headlight sight distance

$$R = \frac{D^2}{2(h + D\tan q)} \qquad (12.5)$$

where

R = minimum. radii sag curve

D = sight distance required (m)

h = mounting height of headlights (m)

q = elevation angle of beam (+ upwards)

The headlight beam is assumed to be directed upwards 1° in calculating the sag curve required.

Table 12.6 Minimum Radii Sag Vertical Curves for Comfort and Head Light Criteria

Design Speed (km/h)	Comfort Criteria		Type S1 Restrictions to Visibility* (h_1=0.75, h_2=0, q=1°)		
	a=0.05g	a=0.1g	Manoeuvre Sight Distance	Stopping Sight Distance	
			R_T=2.0s	R_T=2.5s	R_T=2.0s
50	390	200	660		660
60	570	280	1000		1100
70	770	390	1400		1600
80	1000	500	1900	2400	2100
90	1300	640	2500	3100	2800
100	1600	790	3500†	3900†	
110	1900	950		5000†	
120	2300	1100		6100†	
130	2700	1300		7500†	

* Values in these columns to be used where economically justified.

† The sight distance used to calculate these radii are greater than the range of most headlights (i.e. 120-150m).

NOTES:

1. Values of sight distance used for calculation of minimum radii sag curves are from Chapter 9 Sight Distance.
2. Those values not shown are generally not used.

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Overhead Obstructions

Overhead obstructions such as road or rail overpasses, sign gantries or even overhanging trees may limit the sight distance available on sag vertical curves. With the minimum overhead clearances normally specified for roads, these obstructions would not interfere with minimum stopping sight distance. They may, however, need to be considered with the upper limit of stopping distance (including sight distance to intersections) and overtaking provision.

For Overhead Sight Distance

$$R = \frac{D^2}{2(\sqrt{H - h_1} + \sqrt{H - h_2})^2} \qquad (12.6)$$

where

R = minimum. radii sag curve

D = sight distance required

H = height of overhead obstruction

h_1 = height of eye

h_2 = object cut-off height

Using an eye height of 1.8m and an object cut off height of 0.6m (Commercial vehicle eye height to vehicle tail light - refer Chapter 9) Equation (12.6) becomes:

$$R = \frac{D^2}{2(\sqrt{H - 1.8} + \sqrt{H - 0.6})^2} \qquad (12.7)$$

12.3.5 Reverse Vertical Curves

Reverse vertical curves with common tangent points are considered quite satisfactory and this geometry is often used in grading interchange ramps to achieve the maximum elevation in the shortest acceptable distance. In the case of short radius reverse vertical curves it is necessary to check that the sum of the radial accelerations at the common tangent point does not exceed the tolerable allowance for riding comfort i.e. 0.1g or 0.05g, whichever is appropriate.

A satisfactory buffer length is assumed as equal to 0.1V in metres (V in km/h) but the desirable length is twice this value and is equal to 0.2V in metres. Where less than the required buffer length is available the minimum radii vertical curves are to conform to the following empirical formula.

$$\frac{R(R_1 + R_2)}{R_1 R_2} \leq (1 + a) \qquad (12.8)$$

where

R_1 & R_2 = radii of the two curves being tested

R = minimum radius listed in Table 12.6 (comfort criteria) for absolute or desirable conditions as the case may be.

a = a fraction, being the ratio of the actual length between the TP's of the adopted curves to the normally required buffer length, 0.1Vm (absolute) or 0.2Vm (desirable), as the case may be.

12.3.6 Broken Back/Compound Vertical Curves

Broken back vertical curves consist of two curves, either both sag or both crest, usually of different radii, joined by a short length of straight grade. They should be avoided, particularly in the case of sag curves, and it is usually easy to do so. However, where the length of grade exceeds 0.4Vm (V = design speed in km/h) the curves are not then deemed to be broken-backed. If there is no grade, that is, the tangent points are common, the curves are compound, not broken-backed, and are permitted.

A particular case of a compound curve is the asymmetric vertical curve described in detail in Appendix 12B.

12.4 Procedure for Determination of Minimum Radius Vertical Curves

12.4.1 Minimum Radius Crest Vertical Curves

Appearance Criterion

Use Table 12.4 to determine the maximum changes of grade without a vertical curve and minimum length of vertical curve for satisfactory appearance. Minimum radii at small changes of grade can be calculated using the criteria given in Table 12.4.

Sight Distance Criteria

Step 1 - Use Table 9.2 in Chapter 9 to determine the various types of sight distance suitable for the roadway type.

Step 2 - Use Table 9.4 in Chapter 9 to select the appropriate reaction time for the roadway type.

Step 3 - Use Table 9.1 in Chapter 9 to select the appropriate vertical height parameters.

Step 4 - Read off the minimum radii crest vertical curves from Table 12.5 for the design speed and appropriate types of sight distance from Step 1.

12.4.2 Minimum Radius Sag Curves

Appearance Criterion

The minimum radii sag curves for appearance are the same as found for crest curves (i.e. from Table 12.4).

Comfort Criterion

Use Table 12.6 to determine minimum radii sag curves for comfort based on design speed and maximum rates of vertical acceleration.

Sight Distance Criteria

Step 1 - Use Table 9.2 in Chapter 9 to determine the various types of sight distance suitable for the roadway type.

Step 2 - Use Table 9.4 in Chapter 9 to select the appropriate reaction time for the roadway type.

Step 3 - Read off the minimum radii sag vertical curves from Table 12.6 for the design speed and appropriate types of sight distance from Step 1.

If an overhead obstruction is present, then the minimum radius sag vertical curve based on overhead obstruction criteria from Section 12.3 will need to be calculated.

Example 12.2 illustrates the application of this procedure.

12.5 Grading of Vertical Alignments

The grading of vertical alignments in rural type environments will impact greatly on the safety, operational performance, economics and visual aspects of the proposed road infrastructure.

Besides meeting the coordination requirements addressed in Chapter 10 the vertical alignment should seek the most economical solution within the determining parameters. It is essential that the vertical alignments be developed in conjunction with the other geometric elements, e.g. horizontal alignment and cross sections. Geotechnical testing of existing in situ material is an essential prerequisite to establishing cross section profiles and for economical grading purposes.

12.5.1 Economics in Grading

It is established design practice to achieve the most economical solution when designing a road. A key aspect in achieving this requirement is to seek a balance in earthworks and to reduce the haulage distance from cuts to fills.

Earthworks computer systems identify the net quantities of cut and fill material together with the

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different types of cut material, e.g. rock (non-rippable material), earth (rippable material). They will also produce a mass haul diagram which converts the net cut volumes to embankment volumes. Where a grade line is not constrained by other influences/restrictions, obtaining the most economical grading solution requires the designer to use a mass haul diagram, to enable him/her to:

- balance the earthworks;
- optimise the length of leads from cuts to fill;
- locate job borrow areas; and
- match the areas of spoil to the most appropriate spoil site(s).

The mass haul diagram is also a very useful tool to assist in the costing of earthworks as it identifies:

- the locations of cut to fill leads;
- the location of spoil material;
- the location of borrow requirements;
- the haulage work to be performed.

A construction contractor may also take into account the estimated number of haulage cycles based on bulked/compacted state of the material when preparing a tender.

Geotechnical testing of in-situ material is essential for determining:

- cut slopes/benching details;
- embankment slopes/benching details;
- suitability for the subgrade, i.e. as a working platform for placing the road pavement;
- material bulking when led from cut to fill;
- material compaction when led from cut to fill.

These factors will also influence the fixing of the grade line and the cost of earthworks operations.

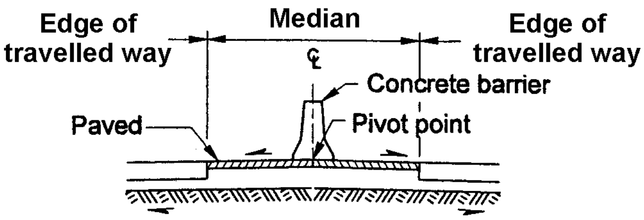
The efficiency of a grade line is only relevant to the section of road concerned. For example, if a section of undulating road 10km long is graded as a single construction project, a particular grading solution will result. If the job were broken up into three separate construction projects, each section would need to be re-graded to achieve the optimum grading solution for that section. This is because the different earthworks characteristics (i.e. cut, fill, borrow, spoil) in the smaller sections will invariably restrict the options for balancing the earthworks. However, it is important that this does not lead to a disjointed result when the total length is considered.

12.5.2 Grading of Dual Carriageways

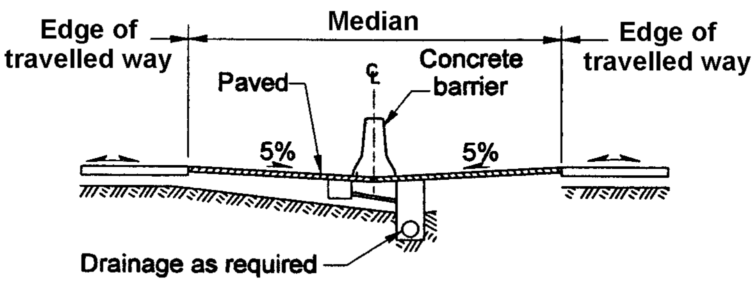
Where medians up to 8m wide are used, the adjacent median shoulder edges should be level or follow the crossfall of the road on curves (Clause 7.7.4). (This approach is also desirable for median widths up to 15m wide where future lane additions are proposed in the median.) This will also assist in the attainment of the flatter batter slopes specified for medians and to provide a consistent median section. It also allows for the adding of an extra lane into the median with standard double-sided concrete barrier installation and still achieves a solution that is safe and economical (see Figure 12.2). Median water discharge intervals are a critical design issue when fixing median profiles.

In urban areas the medians are often constrained due to the available right-of-way width and in many situations there is only room for a double-sided concrete barrier to separate the two opposing carriageways. In this circumstance the grading of each carriageway should be such as to accommodate the concrete barrier without the need to provide for variations in the heights between the two carriageways, i.e. the height of the adjacent shoulder points should for all intents and purposes, be the same.

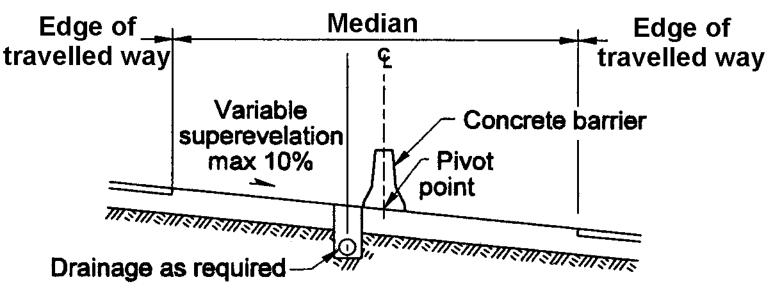
In some circumstances differential grading may be adopted for each carriageway due to economics and/or environmental considerations. In this event non-parallel horizontal alignments may also be employed to cater for the additional median width required to accommodate cut/fill slopes. These features of design are most likely to



Crowned Median

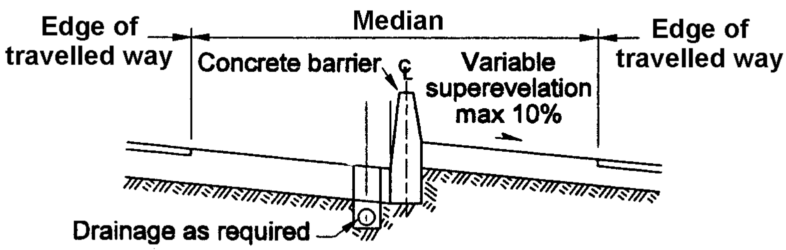


Depressed Median



Single Pivot Point

DESIRABLE GRADING OUTCOMES



Dual Pivot Points

UNDESIRABLE GRADING OUTCOMES

(To be avoided where possible)

Figure 12.2 Dual Carriageways Median Sections occur in rugged terrain and it is therefore important to ensure appropriate consideration is given at this stage for any future carriageway widening, such as adding an additional lane for future overtaking/climbing opportunities or to simply add an extra carriageway lane for capacity purposes.

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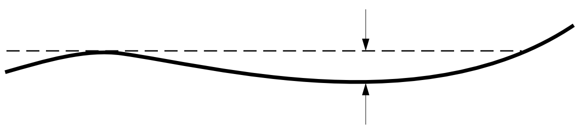
12.6 Hidden Dip Grading

On long lengths of straight alignment, particularly in slightly rolling country, hidden dips should be avoided wherever possible. At times, in periods of high glare, or poor visibility, and because of the foreshortening effect due to the level of the eye, an illusion of apparent continuity of pavement omitting the dip is sometimes created. (See Figure 10.7.)

Hidden dips contribute to overtaking manoeuvre accidents, the overtaking driver being deceived by the view of highway beyond the dip free of opposing vehicles. Even with shallow dips, this type of profile is disconcerting because the driver cannot be sure whether or not there is an oncoming vehicle hidden beyond the crest. This type of profile can be avoided by appropriate horizontal curvature or by more gradual grades made possible by heavier cuts and fills.

It is preferred that the entire pavement surface is visible in these cases. However, if there is no alternative, a maximum depression of 600mm below the driver's line of sight may be tolerated. Guidance on the limited depth of the depression gives confidence to drivers. Road edge guide posts at close intervals would provide such guidance.



12.7 Drainage Issues

12.7.1 Apex

On crest vertical curves, the slope of the table drain near the apex reduces to below 0.5% for a length dependent on the radius of the curve. On large radius curves, this length can be substantial and ponding of water may occur. To overcome this, the length of table drain with a slope less than 0.5% should be limited to about 60m (corresponds to a radius of 6000m).

Where the curve radius is greater than this, action has to be taken to provide adequate drainage at the apex. Possible solutions include reducing the depth of the table drain at the crest (the reduced volume of water can be accommodated but the drainage of the pavement has to be considered), and lining the table drain with concrete. Adjusting the grade of the table drain relative to the centre line grade over the full length of the curve may also be considered. In extreme cases, it may be necessary to provide underground drainage to achieve a satisfactory solution.

If the road is kerbed, careful attention to the design of inlets, their spacing and the underground drainage system is required. It is essential that water is not allowed to accumulate on the pavement and create an aquaplaning problem.

12.7.2 Sags

Drainage of the bottom of sag curves has to be considered. Where the road is not kerbed, this should be a simple matter and is generally not a problem. However, where kerbs are used, it is essential that sufficient capacity of the gully inlets and underground drainage system be provided to limit ponding of water on the surface in accordance with the criteria set out in the Drainage Manual. It is also important that any accumulation of water does not create an aquaplaning problem.

12.8 Floodways

The longitudinal grade on the approach to floodways must be carefully designed to:

- avoid discomfort to the occupants of vehicles;
- provide stopping sight distance to the surface of the water in a short floodway; and
- ensure that drivers are not misled regarding the extent and depth of the floodway.

The vertical curves should be designed in accordance with the comfort criteria described in 12.3.4. For short floodways, it is important that drivers can see the presence of water on the road and the sight distance should be checked to ensure that stopping distance is achieved to the height of the water surface at a depth of 150mm.

In flat country, the presence of the floodway must be obvious to the driver and a relatively short, sharp entrance to the floodway section (within the comfort criterion) should be provided. It is also essential to avoid more than one level in a floodway. That is, once the driver has entered the floodway with water across it, there must be no deeper water at some point further along the floodway. This type of design is misleading to drivers and can result in a dangerous situation.

Details of the appropriate approach grading to floodways are given in Figures 12.3 and 12.4.

12.9 Grading Over Grids and Pipes in Flat Country

In flat country where the grade of the road is close to the natural ground level, grids and drainage pipes cannot be effectively placed such that they are below the normal grade line of the road. To do so would create drainage problems as the base of the grid and the invert of the pipe would be below the surrounding countryside. In these cases, the road has to be graded over the grid or pipe to allow them to function and to provide adequate cover to the pipe. Figure 12.5 provides details of the method for doing this.

12.10 Grading at Railway Level Crossings

Grading at railway level crossings requires special attention to ensure a safe and comfortable crossing. The location of the crossing has to be carefully considered to ensure as close a coincidence of grades and levels as possible. It may be necessary to relocate either or both of the road and rail to obtain a satisfactory result in some cases.

Details of the approach to be taken are given in Chapter 21 “Railway and Cane Railway Level Crossings”.

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Queensland Department of Main Roads (2001): Road Drainage Design Manual.

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Relationship to Other Chapters

- Close relationship with Chapter 6 – Speed Parameters;
- Cross section is affected by vertical alignment issues (Chapter 7);
- Sight distance requirements are defined in Chapter 9;
- Has to be read in conjunction with Chapters 10 and 11;
- Elements of Chapters 4, 13, 14, 15, 16, 20, 21 and 22 require information from this chapter.

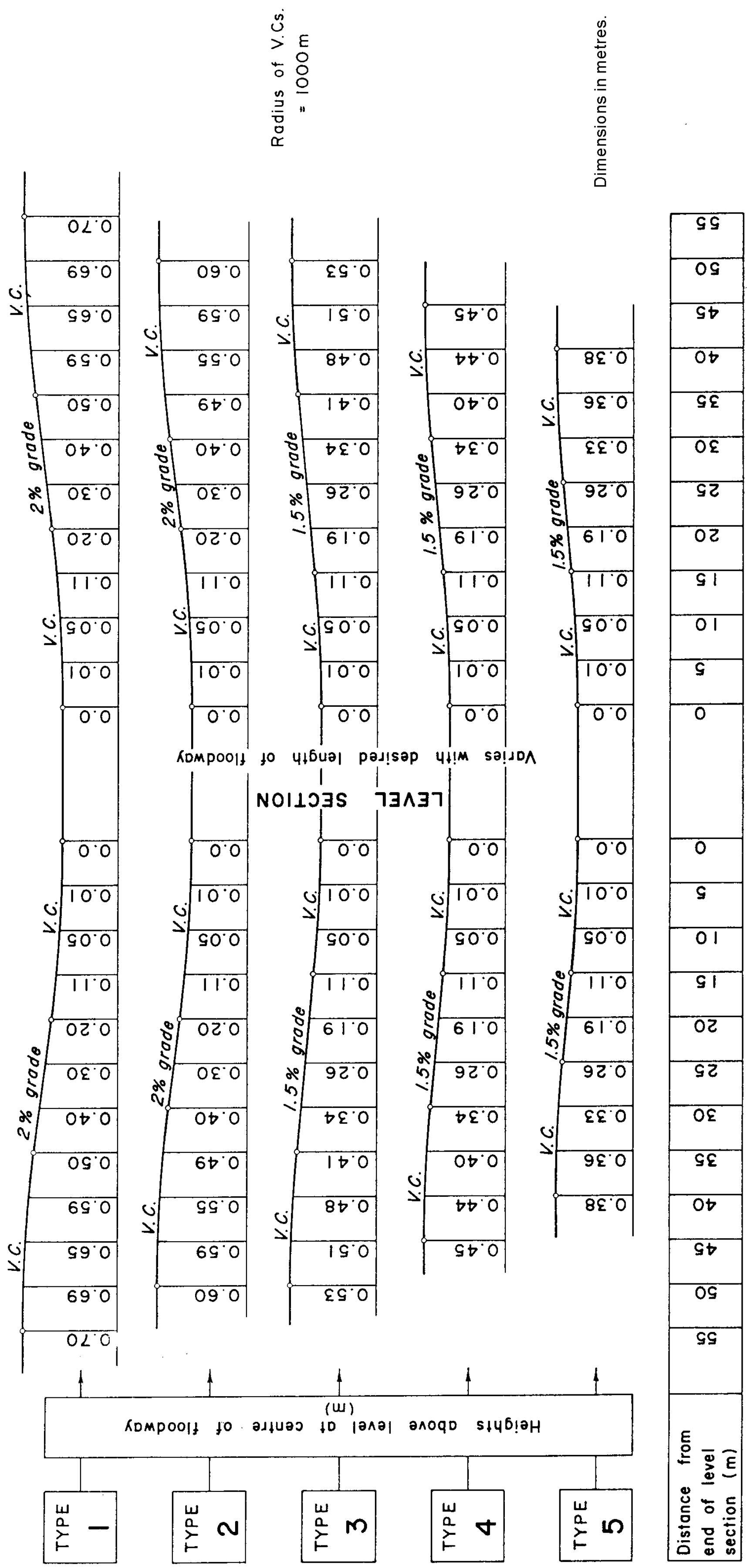

Figure 12.3 Standard Floodways

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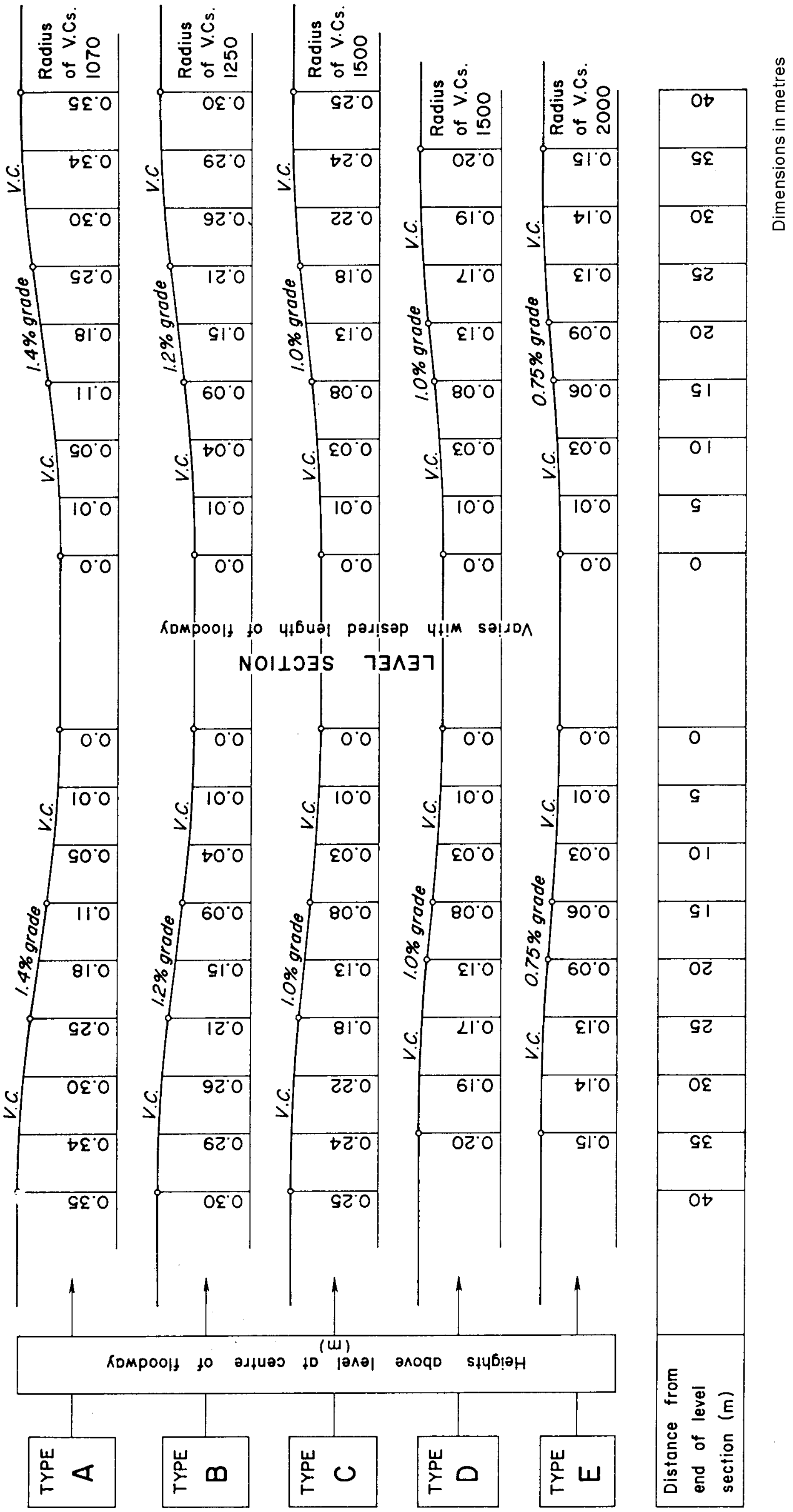
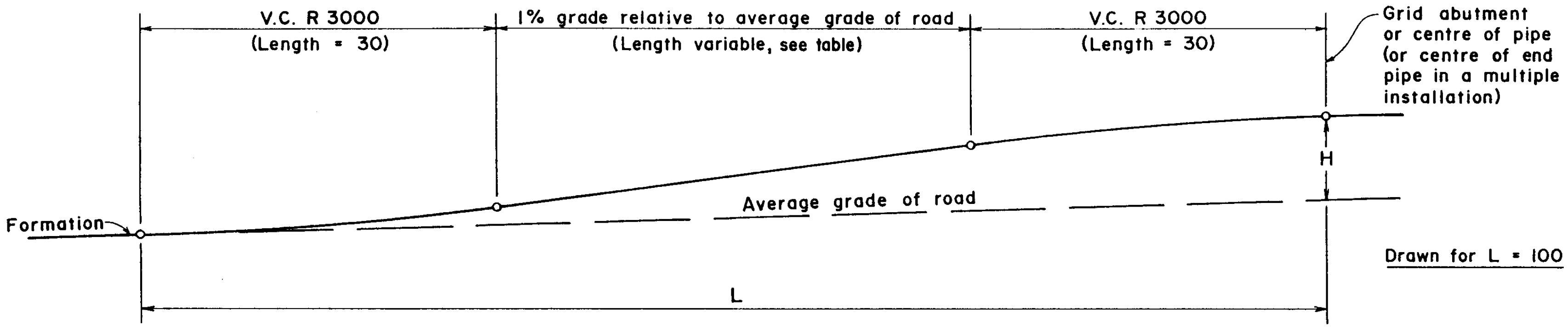


Figure 12.4 Standard Floodways in Low Formation



HALF LONGITUDINAL SECTION

TABLE OF HEIGHTS ABOVE AVERAGE GRADE OF ROAD

Height H	Length L	Distance from grid or pipe									Length of grade
		10	20	30	40	50	60	70	80	90	
0.7	100	0.68	0.63	0.55	0.45	0.35	0.25	0.15	0.07	0.02	40
0.6	90	0.58	0.53	0.45	0.35	0.25	0.15	0.07	0.02	–	30
0.5	80	0.48	0.43	0.35	0.25	0.15	0.07	0.02	–	–	20
0.4	70	0.38	0.33	0.25	0.15	0.07	0.02	–	–	–	10
0.3	60	0.28	0.23	0.15	0.07	0.02	–	–	–	–	0

NOTES

1. The depth of covering over the crown of a pipe at the crown of the paving shall, wherever possible, be not less than 0.6m, but in no case shall it be less than 0.4m at the crown of the paving or 0.25m at the outside edge of the shoulder.

2. The soffit of a pipe must not be above th; average grade line of the road.

Figure 12.5 Method of Grading Over Grids and Pipes in Flat Country (Dimensions in metres)

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Example 12A

Problem

Find the minimum radius crest vertical curve for following case:

- Two lane, two way road
- Design speed = 100 km/h
- Reaction time = 2.5 secs
- Use single vehicle stopping sight distance to an object of cut-off height 0.2 m

Solution

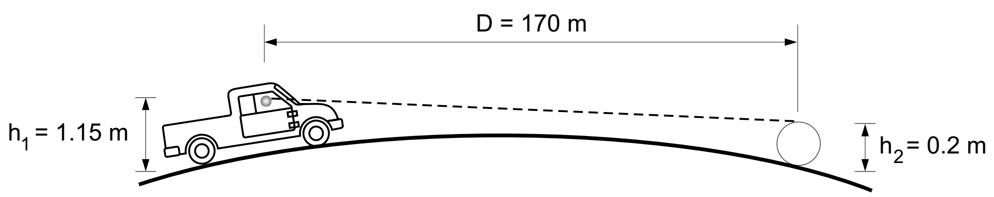
Using Table 9.7 for a design speed of 100 km/h and R_T=2.5s, stopping sight distance = 170 m. From Table 9.1, height of eye for a passenger car 'h_1' = 1.15 m and object cut-off height 'h_2' = 0.2m.

Using Equation 12.2 where the length of vertical crest curve is greater than the sight distance, the following applies:

$$R = \frac{D^2}{2(\sqrt{h_1} + \sqrt{h_2})^2}$$

$$R = \frac{170^2}{2(\sqrt{1.15} + \sqrt{0.2})^2} = 6258 \quad \text{say} \quad 6260$$

$\therefore$ minimum radius crest vertical curve = 6260 m



Example 12B

Problem

Determine the minimum radius crest and sag vertical curves for the given roadway.

- two lane, two way undivided road.
- design speed - 90km/h
- rural road with alerted driving conditions (unlit roadway)

Solution

Minimum Radius Crest Vertical Curves

Appearance Criterion

Interpolating from Table 12.4, for a design speed of 90km/h, maximum grade change without vertical curve = 0.5%. Minimum length of vertical curve from Table 12.4 = say 70 - 90m. Minimum radii at small changes of grade can be calculated using this criterion.

Sight Distance Criteria

Using Table 9.2 of Chapter 9, for a two lane, two way, undivided road, the following apply:

1. Manoeuvre sight distance is the absolute minimum (only use in extremely isolated or constrained cases - minimum carriageway widths apply).
2. Stopping sight distance is the general minimum.
3. Overtaking sight distance is desirable.

Using Table 9.4 of Chapter 9, for a rural road with alerted driving conditions, R_T = 2.0s.

From Table 9.1 of Chapter 9, an object cut-off height = 0.2m.

From Table 12.5, for a design speed of 90km/h with R_T = 2.0s and an object cut-off height = 0.2m, the following minimum radii crest vertical curves are applicable for the various sight distance types:

(1) Manoeuvre sight distance, for Type C1 restrictions to visibility (lit roadway), radius = 3100m

(2) Manoeuvre sight distance, for Type C2 restrictions to visibility (unlit roadway), radius = 4180m

(3) Stopping sight distance, for Type C1 restrictions to visibility (lit roadway), radius = 3700m

(4) Stopping sight distance, for Type C2 restrictions to visibility (unlit roadway), radius = 4900m

(5) Overtaking. It is generally not practicable to provide overtaking sight distance over crest vertical curves because of the length of curve required.

From the above criteria, the preferred minimum radius crest vertical curve is 4900m. This allows stopping sight distance for 90 km/h based on headlight criteria. Where this cannot be economically justified, 3700m would be the general minimum. In extremely isolated or constrained locations, 3100m may be used provided the minimum pavement widths in Section 9.3.1 are provided.

Minimum Radii Sag Vertical Curves

Appearance Criterion

Same radii as for appearance of crest vertical curves.

Comfort Criterion

From Table 12.6, for a design speed of 90km/h, and a = 0.05g (normal for rural road), minimum radius sag vertical curve = 1300m. In extremely isolated or constrained situations, a minimum radius sag vertical curve = 640m may be used (a = 0.1g).

Sight Distance Criteria

Using Table 9.4 of Chapter 9, for a rural road with alerted driving conditions, R_T = 2.0s.

From Table 12.6, for a design speed of 90km/h and R_T=2.0s, the following minimum radii sag vertical curves are applicable for the various sight

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distance types:

- Manoeuvre sight distance, radius = 9600m
- Stopping sight distance, radius = 11300m

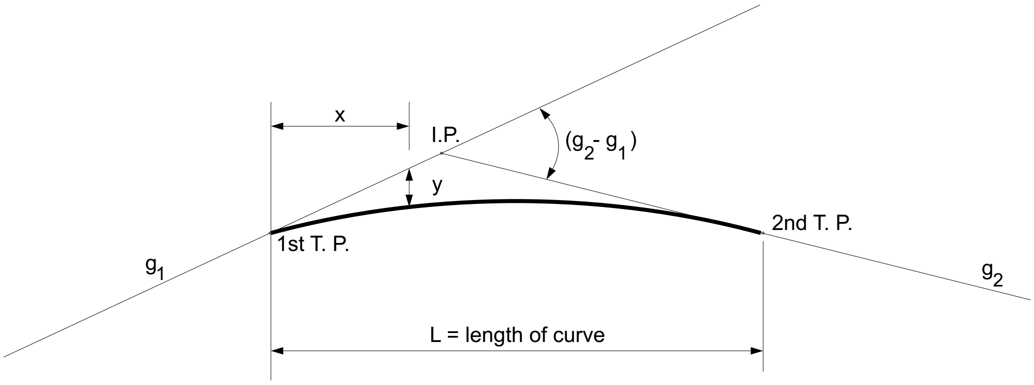
From the above criteria, the desirable minimum radius sag vertical curve is 11300m to provide headlight criteria. Usually, such a large radius sag vertical curve cannot be justified unless on a high standard road in flatter terrain. A general minimum of 1300m is applicable (comfort criteria).

Appendix 12A

Derivation of Vertical Curve Parabola Formulae

g_1 = 1st grade in % (+VE if rising)

g_2 = 2nd grade in % (-VE if falling)



Parabola equation: $y = kx^2$ *(1)*

where

y = offset of parabola down from grade line (m)

k = constant

x = horizontal distance from tangent point to the offset point (m)

$$\text{gradient of parabola} = \frac{dy}{dx} = 2kx \qquad (2)$$

$$@\ x = L,\ \text{gradient} \quad \frac{g_2 - g_1}{100} \ \text{m/m} \qquad (3)$$

substitute (3) into (2)

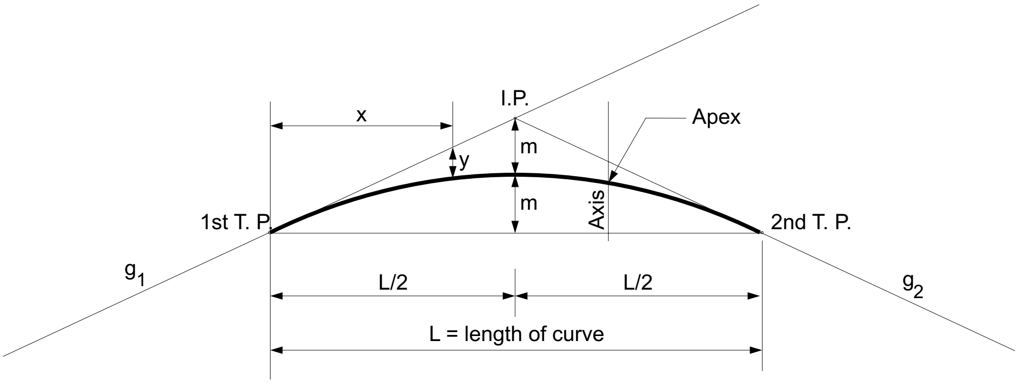
$$\frac{dy}{dx} = 2KL = \frac{g_2 - g_1}{100}$$

$$K = \frac{g_1 - g_2}{200L} \qquad (4)$$

substitute (4) into (1)

$$y = \frac{(g_2 - g_1)x^2}{200L} \qquad (5)$$

Derivation of Equivalent Radius



Length equally spaced about the I.P.

$$y = \frac{(g_2 - g_1)x^2}{200L} \qquad (5)$$

$$\text{gradient} \quad \frac{dy}{dx} = \frac{(g_2 - g_1)x}{100L} \qquad (6)$$

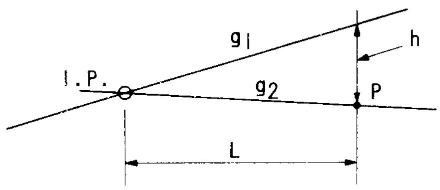
$$\text{rate of change of gradient} \quad \frac{d^2y}{dx^2} = \frac{g_2 - g_1}{100L}$$

$$\text{equivalent radius} \quad (R) = \frac{1}{\frac{d^2y}{dx^2}} = \text{radius @ Apex}$$

$$R = \frac{100L}{g_2 - g_1} \qquad (7)$$

Solutions to Common Vertical Geometry Problems

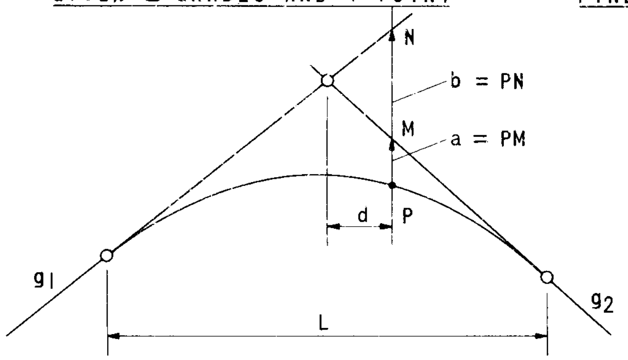
GIVEN 2 GRADES **FIND I.P. CHAINAGE**



At any suitable chainage P, calculate levels on the two grades and find difference h.

$$\text{Then } L = \frac{100h}{(g_1 - g_2)}$$

GIVEN 2 GRADES AND 1 POINT **FIND V.C.**

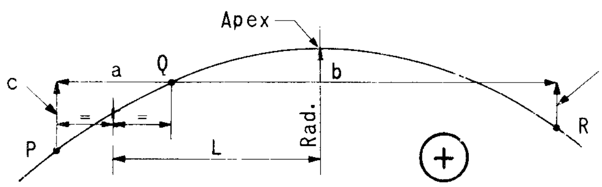


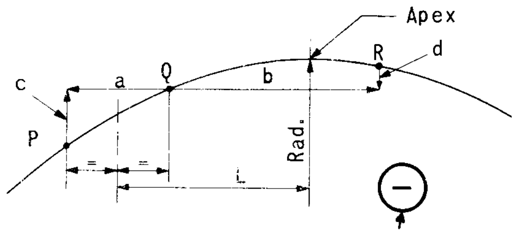
Given g_1 g_2 d and point P

Calculate levels on the two grades and find -
PM = a
PN = b

$$\text{Then } L = \frac{\sqrt{b} + \sqrt{a}}{\sqrt{b} - \sqrt{a}} \cdot 2d$$

GIVEN 3 POINTS **FIND V.C.**





1. Select inner point in chainage
   Calculate lths. a, b and hts. c, d (neglect signs)

   $$\text{Then } R = \frac{ab(a + b)}{2(ad \pm bc)}$$

   Use - sign when selected inner point is also the inner point in height, as sketch above. Ignore final sign of denominator.

2. To find chainage of apex:
   Calculate chainage midway P to Q

   $$\text{Then } L = \frac{cR}{a} \quad (R = \text{radius of V.C.})$$

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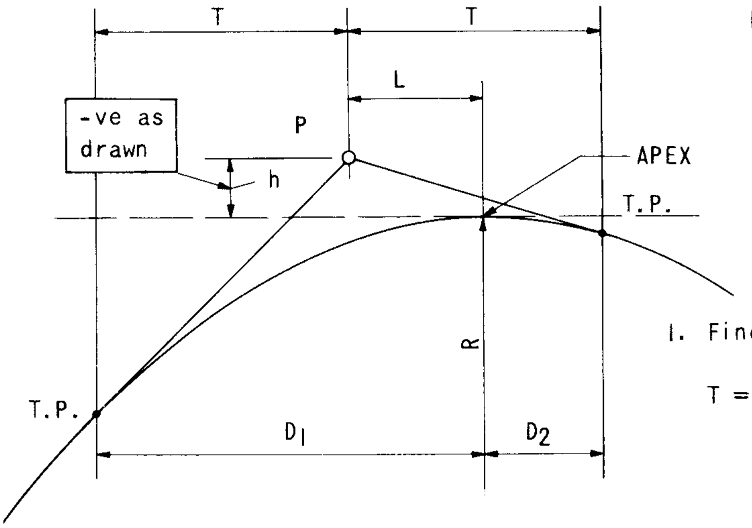
GIVEN V.C. AND POINT OUTSIDE — FIND TANGENT



R, L and T always +ve

h +ve if on same side of apex tangent as V.C.

-ve if on opposite side of apex tangent as V.C. (as drawn)

1. Find length of tangent

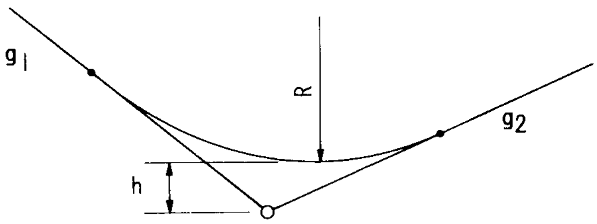
$T = \sqrt{L^2 - 2Rh}$ ← watch sign of h

There will always be two tangents.

2. Find length from T.P. to apex, D_1 or D_2, from $D = L \pm T$ (Select sign by inspection to suit tangent required)

3. Find grade of tangent: $g\% = \frac{100D}{R}$ (Select sign of g by inspection)

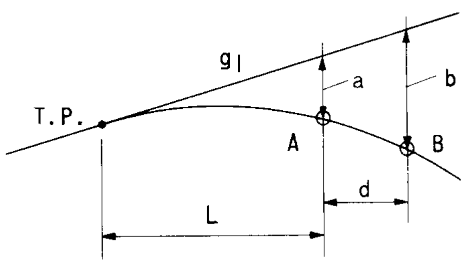
GIVEN 2 GRADES AND R.L. OF APEX — FIND V.C.



Find radius R:

$R = \frac{20000h}{g_2 - g_1}$ (Neglect signs)

GIVEN 1 GRADE AND 2 POINTS — FIND V.C.

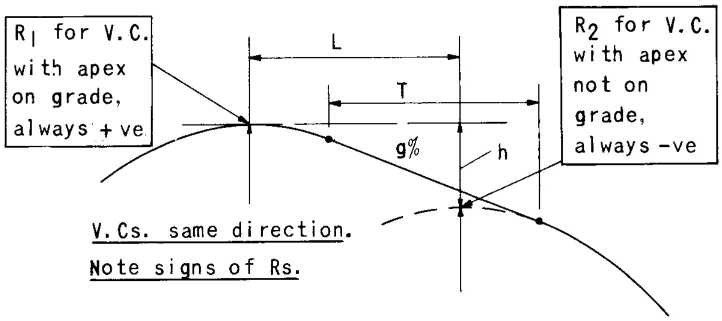


Calculate heights a and b

Then $L = \frac{\sqrt{a}}{\sqrt{b} - \sqrt{a}} \cdot d$

and $R = \frac{L^2}{2a}$

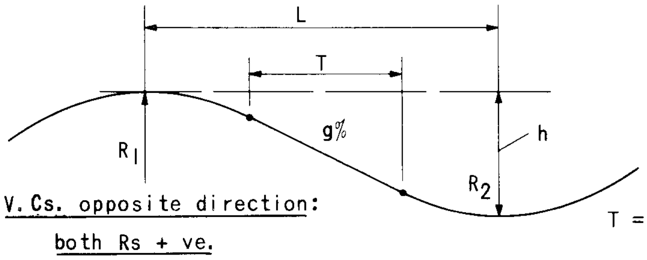
GIVEN 2 V.Cs. FIND COMMON TANGENT



h always + ve

R_1 always V.C. with apex on grade and always + ve

R_2 may be -ve if apex of V.C. is not on grade



CASE 1:- When $(R_1 + R_2) \neq 0$

(i) Find length of tangent T:

$$T = \sqrt{L^2 - 2h(R_1 + R_2)}$$

Watch sign of $(R_1 + R_2)$ when R_2 is -ve.

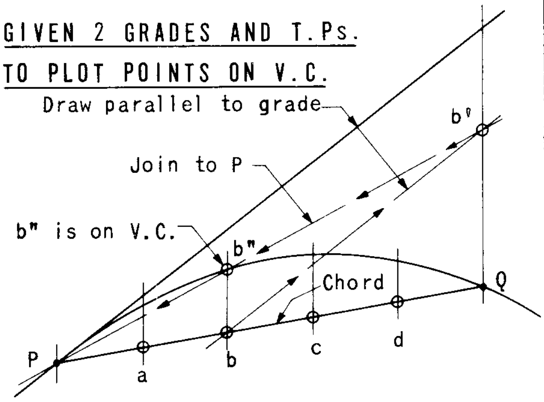
(ii) Find $D = (L-T)$ Ignore resulting sign.

(iii) Find $g\% = \dfrac{100D}{(R_1 + R_2)}$

When R_2 is -ve and $R_2 > R_1$, the resulting -ve sign of (R_1+R_2) may be ignored, but only in this step.

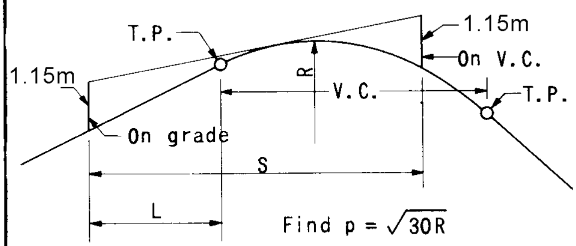
CASE 2:- When $(R_1 + R_2) = 0$
Use simple formula: $g\% = \dfrac{100h}{L}$

GIVEN 2 GRADES AND T.Ps. TO PLOT POINTS ON V.C.



Mark series of points a'b'c'... on Q-ordinate and join back to P.
Points a"b"c"... are on V.C.

GIVEN V.C. AND GRADES. FIND POINT WHERE SIGHT DISTANCE S CUTS OUT



Find $p = \sqrt{30R}$

L = distance T.P. to reqd. pt.

Then $L = \sqrt{S(S - p)}$

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Appendix 12B

The Asymmetric Vertical Curve

The Asymmetric Vertical Curve is a compound vertical curve that is fitted between two grade lines as shown in Figures 12B.1 and 12B.2. The two component vertical curves are standard parabolic curves (to each of which the general VC equations apply) that are tangential to each other. Also, in order to complete the definition of the two component vertical curves (since otherwise an infinite number of combinations are possible), the components are defined by the length l_1 and l_2 each side of the grade intersection point as shown. This also means that the common tangent point (J) between the two component vertical curves is located on the vertical ordinate through the intersection point and that points C, J and D are collinear.

From these "defining properties", the following properties may also be derived:

(a) The grade at J = the grade of line AB (also a property of the standard symmetrical VC).

(b) The asymmetric VC passes through point J such that IJ=JM (also a property of the standard symmetrical VC).

(c) $$\frac{\text{Change\_of\_grade\_over\_}l_1}{\text{Change\_of\_grade\_over\_}l_2} = \frac{l_2}{l_1}$$

To show that the grade at J = the grade of line AB:

The grade at J is the line CJD in 12B.1 and 12B.2 by definition with C being the intersection point for the VC with length l_1 and D being the intersection point for the VC with length l_2.

$\therefore$ for the parabolic curve of length l_1

$$CK = \frac{AN}{2}$$

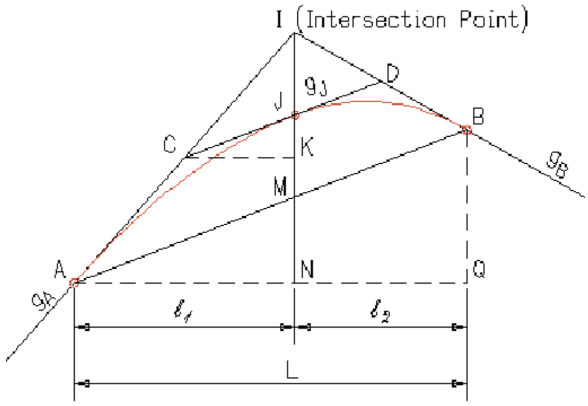

Figure 12B.1 Typical Crest Asymmetric Vertical Curve

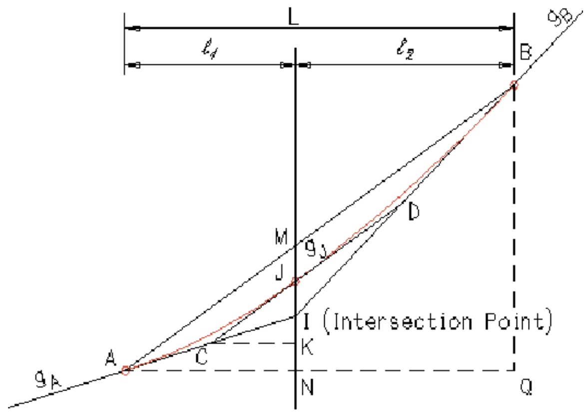

Figure 12B.2 Typical Sag Asymmetric Vertical Curve

But triangles ICK and IAB are similar triangles (note CK || AN)

$$\therefore IC = \frac{IA}{2} = AC$$

Similarly, ID = DB

$\therefore$ line CD cuts sides IA and IB of triangle IAB proportionately

$\therefore$ line CJD || line AMB by geometric theorem [Euclid VI. 2] and triangles ICD and IAB are similar.

hence grade at J = the grade of line AB.

To show that the asymmetric VC passes through point J such that IJ = JM:

In triangles ICJ and IAM, CJ || AM (see above) and are therefore similar.

And IC = CA (see above)

$\therefore$ IJ = JM

Proof that

$$\frac{Change\_of\_grade\_over\_l_1}{Change\_of\_grade\_over\_l_2} = \frac{l_2}{l_1}$$

From Figure 12B.2,
Grade at J = grade of AB

$$= \frac{(g_A l_1 + g_B l_2)}{(l_1 + l_2)}$$

$$= \frac{(g_A l_1 + g_B l_2)}{L} = \frac{g_A l_1}{L} + \frac{g_B l_2}{L}$$

$\therefore$ change over

$$VCl_1 = \frac{g_A l_1}{L} + \frac{g_B l_2}{L} - g_A$$

$$= \frac{g_A l_1}{L} + \frac{g_B l_2}{L} - \frac{g_A L}{L}$$

$$= \frac{g_A l_1}{L} + \frac{g_B l_2}{L} - \frac{g_A (l_1 + l_2)}{L}$$

$$= \frac{g_A l_1}{L} + \frac{g_B l_2}{L} - \frac{g_A l_1}{L} - \frac{g_A l_2}{L}$$

$$= \frac{l_2}{L}(g_B - g_A)$$

and change over

$$VCl_2 = g_B - \frac{g_A l_1}{L} - \frac{g_B l_2}{L}$$

$$= \frac{g_B L}{L} - \frac{g_A l_1}{L} - \frac{g_B l_2}{L}$$

$$= \frac{g_B (l_1 + l_2)}{L} - \frac{g_A l_1}{L} - \frac{g_B l_2}{L}$$

$$= \frac{g_B l_1}{L} + \frac{g_B l_2}{L} - \frac{g_A l_1}{L} - \frac{g_B l_2}{L}$$

$$= \frac{l_1}{L}(g_B - g_A)$$

$$\therefore \frac{Change\_of\_grade\_over\_l_1}{Change\_of\_grade\_over\_l_2} =$$

$$= \frac{l_2 / L(g_B - g_A)}{l_1 / L(g_B - g_A)} = \frac{l_2}{l_1}$$

Derivation of the equation for the offset to the VC at the Intersection Point (m):

From Figure 12B.2,

$$m = MJ = IJ = \frac{IM}{2} = \frac{MN - IN}{2}$$

$$MN = \frac{(g_A l_1 + g_B l_2)}{L} l_1$$

(by proportion from similar triangles AMN and ABQ)

$IN = g_A l_1$

$$\therefore m = \frac{\frac{(g_A l_1 + g_B l_2)}{L} l_1 - g_A l_1}{2}$$

$$= \frac{l_1}{2L}((g_A l_1 + g_B l_2) - g_A L)$$

$$= \frac{l_1}{2L}(g_A l_1 + g_B l_2 - g_A (l_1 + l_2))$$

$$= \frac{l_1}{2L}(g_A l_1 + g_B l_2 - g_A l_1 - g_A l_2)$$

$$= \frac{l_1 l_2 (g_B - g_A)}{2L}$$

Use of the Asymmetric Vertical Curve

The aim of the asymmetric vertical curve is to use a compound vertical curve that better fits a constrained situation yet is still relatively simple to calculate; especially in times prior to the availability of electronic calculators and computers. However, in cases where a compound vertical curve is warranted, a better solution may be found by fixing a common tangent point (and hence common grade line) away from the intersection point of grades g_A and g_B.

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Summary of General VC Equations (grades in "tan form"):

1. When L is defining parameter.

$$y = y_A + g_A x + \frac{(g_B - g_A)}{2L} x^2$$

2. When apex radius, R is defining parameter:

$$y = y_A + g_A x + \frac{x^2}{2R}$$

3. When change of grade/unit length, is defining parameter:

$$y = y_A + g_A x + \frac{\Delta g x^2}{2}$$

4. When offset to VC at IP, m is defining parameter:

$$y = y_A + g_A x + m\left(\frac{x}{L/2}\right)^2$$